Accurate Methods for Fluid Flow: Multipoint Flux

Scientific Achievement

- Development of accurate, locally conservative, multiscale discretizations for multiphase flow on complex geometries.

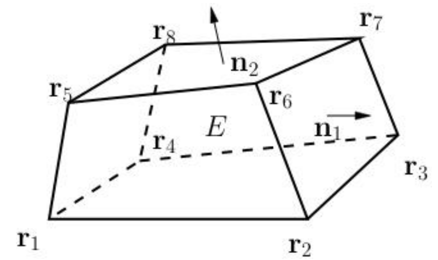
Significance and Impact

- Can handle non-matching grids, full tensors, simplicial elements and distorted hexahedra; easy to implement.

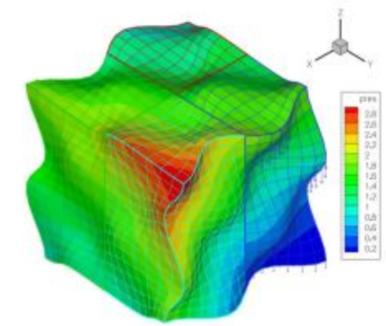
Research Details

- Algorithm based on mixed finite elements; rigorous error estimates derived.
- Results extended to multiphase flow with gravity and capillary pressure curves.
- Modeled Frio CO_2 injection site.
- Can model nonplanar faults and fracture interfaces.

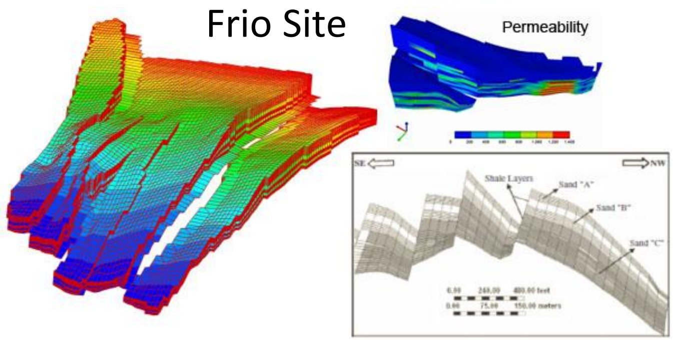
Distorted Hexahedra



Multiscale Discretizations



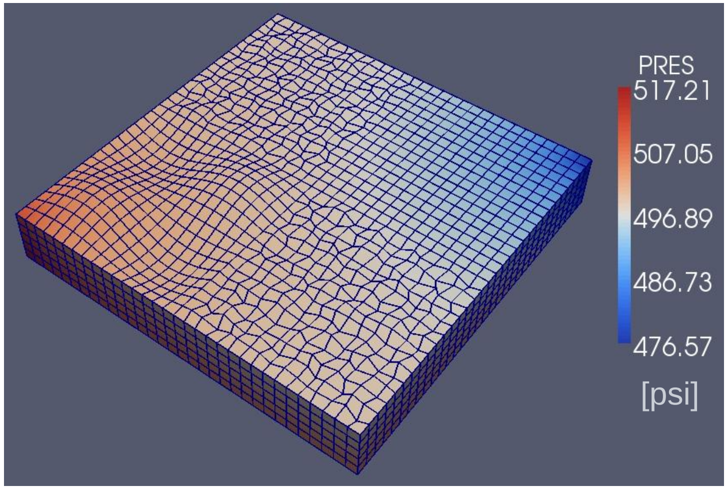
Frio Site



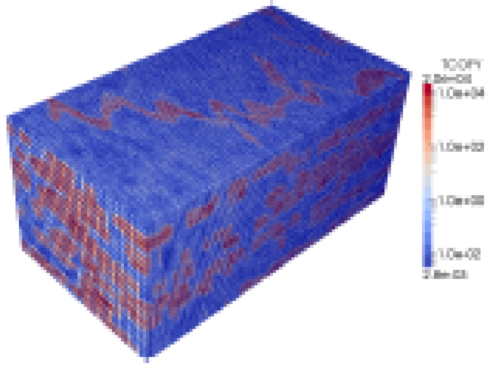
Solver Performance: Multipoint Flux Method

Coupled Symmetric and Non-Symmetric MFMFE Methods

- Symmetric method: for nearly cubic elements
- Non-symmetric method: for highly distorted hexahedral elements



SPE 10 permeability on highly perturbed hexahedral mesh with 1.1M elements.



Solver Performances for SPE 10 Benchmark

Symmetric multipoint flux

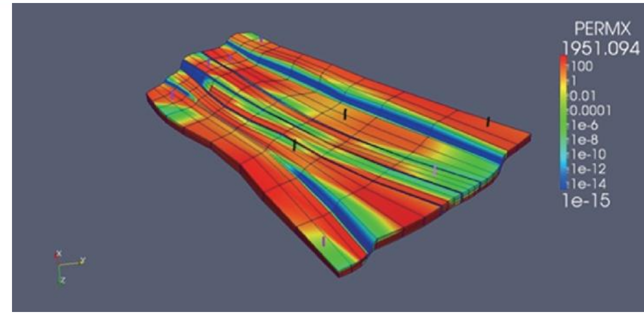
Solver	Iterations
HYPRE	27
SAMG	34
FASP	14
Trilinos ML	21-28

Non-symmetric multipoint flux

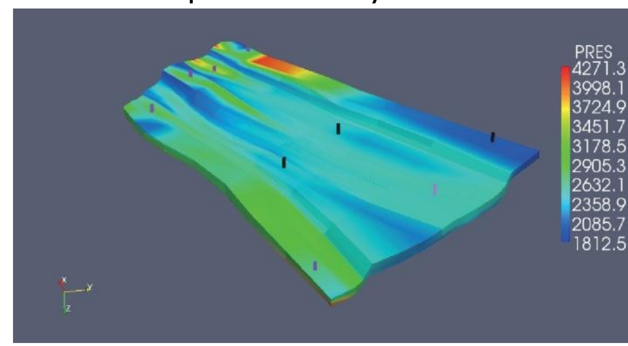
Solver	Iterations
HYPRE	42
SAMG	61
FASP	25
Trilinos ML	23-29

Coupled Poroelasticity on a General Hexahedral Grid

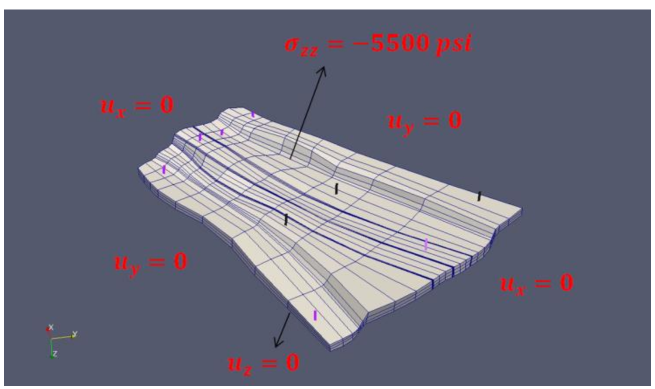
PARAMETER	QUANTITY	VALUE
x	x coordinate	$\in$ (-10.4, 8561.6) ft
y	y coordinate	$\in$ (68.8, 8822.9) ft
z	z coordinate	$\in$ (3796.9, 5436.2) ft
T	total simulation time	7.0 day
ΔT	time step size	0.1 day
P_0	initial pressure	hydrostatic
ρ_0	reference fluid density	$56\ lb_m/ft^3$
η	fluid viscosity	1 cp
c_f	fluid compressibility	$4.0 \times 10^{-7}\ psi^{-1}$
ϕ	initial porosity	0.2
k_{xx}, k_{yy}	horizontal permeability	$\in (1.0 \times 10^{-15}, 1592)$ md
k_{zz}	vertical permeability	$0.1\ k_{xx}$
N_{inj}	number of injection wells	6
BHP_{inj}	bottom hole pressure of injection wells	$\in$ (3300, 4400) psi
N_{prod}	number of production wells	3
BHP_{prod}	bottom hole pressure of production wells	2000 psi
σ_{zz}	vertical stress on reservoir top surface	-5500 psi
E	Young's modulus	1.0×10^6 psi
ν	Poisson's ratio	0.3
ρ_s	rock density	$165\ lb_m/ft^3$



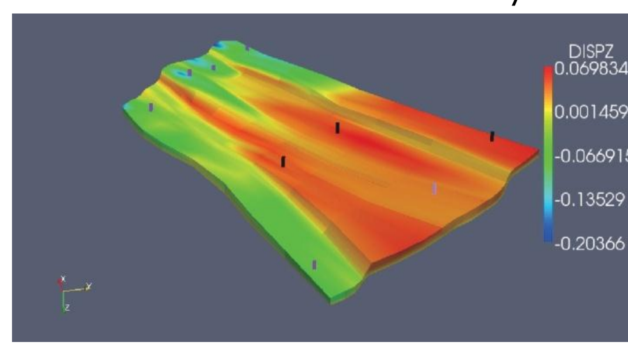
X-permeability Profile



Fluid Pressure at 7.0 Days



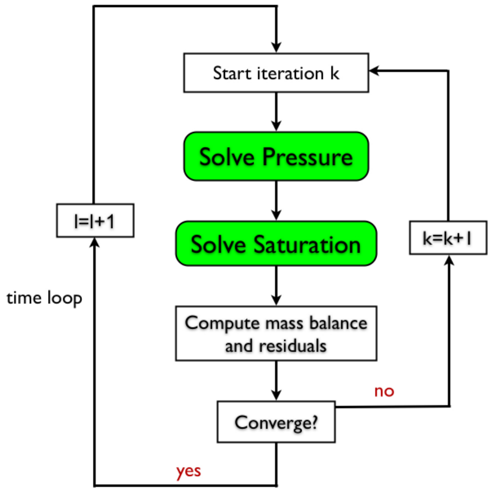
Mechanics Boundary Condition



Vertical Displacement at 7.0 Days

Modeling Capillarity with the Multipoint Flux Method

- Iterative coupling IMPES Scheme



- Brooks-Corey Capillary Pressure

$$p_c(s_e) = p_d s_e^{-\frac{1}{\lambda}}$$

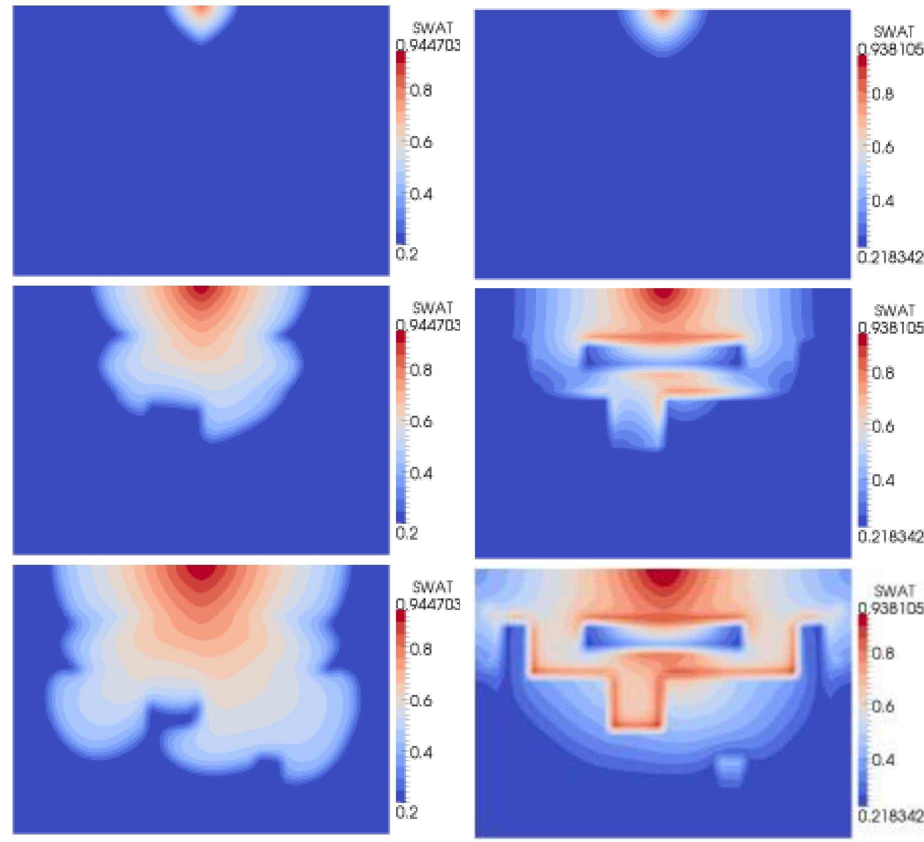
Media type	p_d	λ
type 1	135	2.49
type 2	37.7	3.86

$$s_e = \frac{s - s_{rw}}{1 - s_{rw} - s_{rn}}$$

$k_{rw} = 0.7 s_e^2$ $\quad k_{rn} = 0.5(1 - s_e)^2$

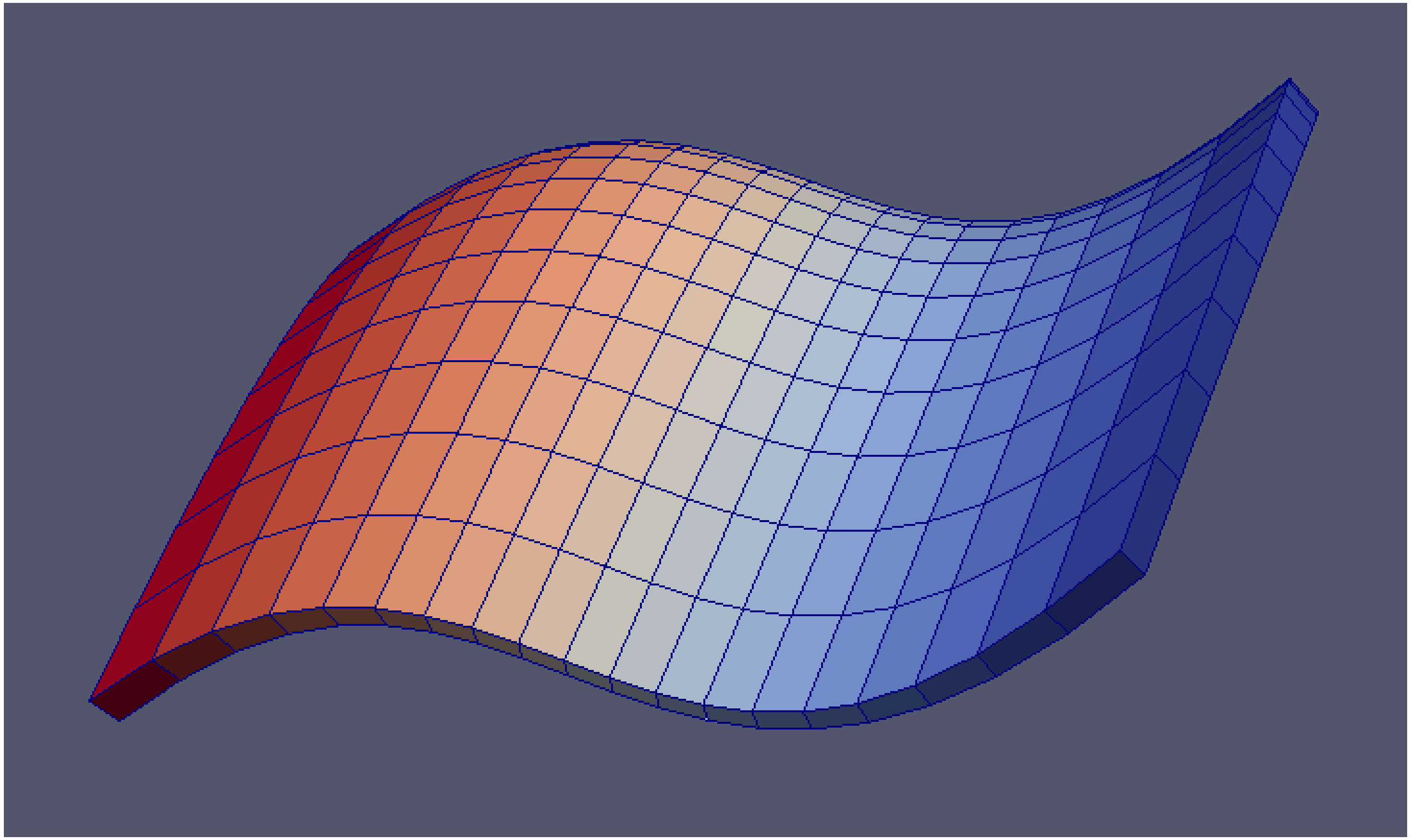
$s_{rw} = 0.2 \quad s_{rn} = 0.05$

Water Saturation without capillarity | Water Saturation with capillarity



Using NURBS to represent nonplanar interfaces

Example of using a Non-Uniform Rational B-Splines (NURBS) to represent nonplanar faults and fractures



References

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