# Accurate Methods for Fluid Flow: Multipoint Flux

## Scientific Achievement

- Development of accurate, locally conservative, multiscale discretizations for multiphase flow on complex geometries.

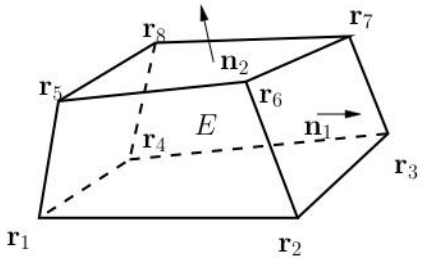
## Significance and Impact

- Can handle non-matching grids, full tensors, simplicial elements and distorted hexahedra; easy to implement.

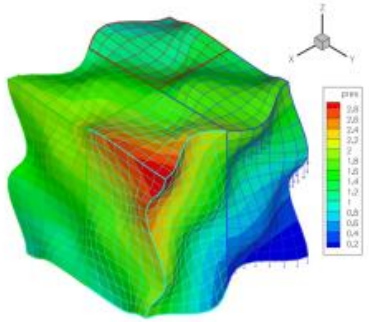
## Research Details

- Algorithm based on mixed finite elements; rigorous error estimates derived.
- Results extended to multiphase flow with gravity and capillary pressure curves.
- Modeled Frio $CO_2$ injection site.
- Can model nonplanar faults and fracture interfaces.

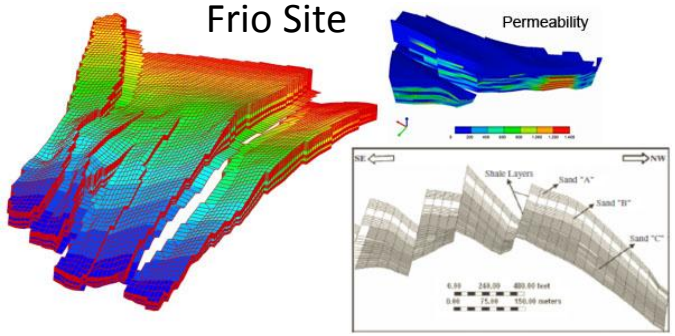
Distorted Hexahedra




Multiscale Discretizations



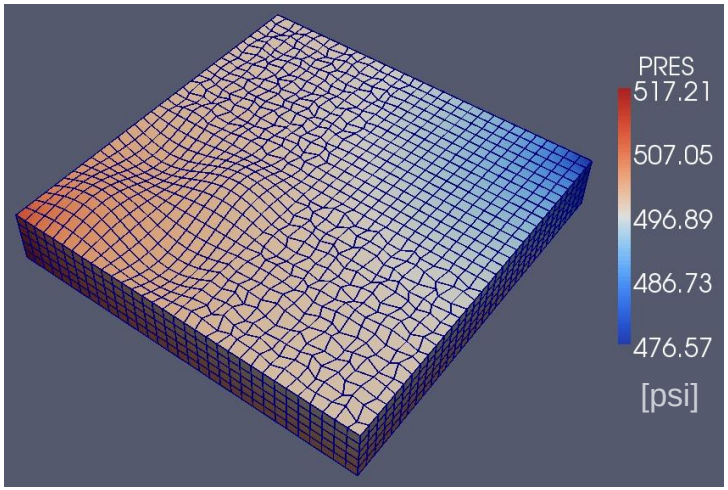
Frio Site



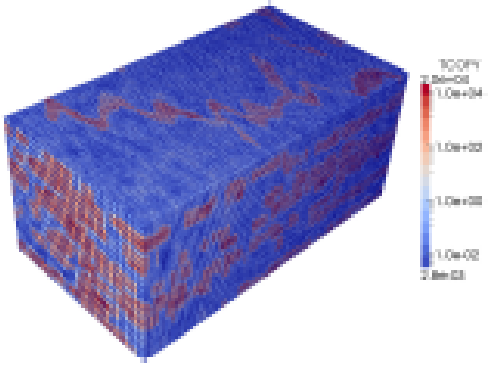
# Solver Performance: Multipoint Flux Method

## Coupled Symmetric and Non-Symmetric MFMFE Methods

- Symmetric method: for nearly cubic elements
- Non-symmetric method: for highly distorted hexahedral elements




SPE 10 permeability on highly perturbed hexahedral mesh with 1.1M elements.



## Solver Performances for SPE 10 Benchmark

Symmetric multipoint flux

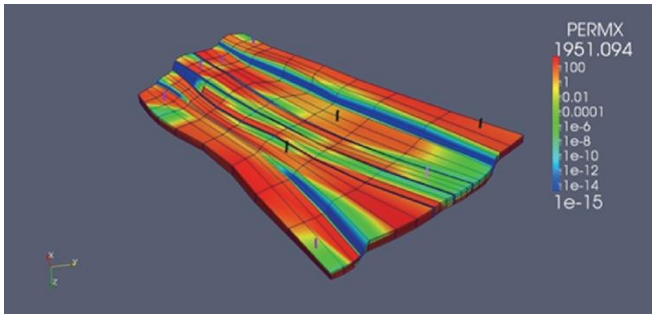
| Solver | Iterations |
|---|---|
| HYPRE | 27 |
| SAMG | 34 |
| FASP | 14 |
| Trilinos ML | 21-28 |

Non-symmetric multipoint flux

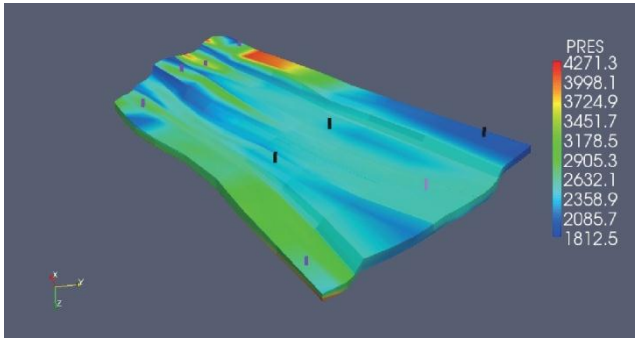
| Solver | Iterations |
|---|---|
| HYPRE | 42 |
| SAMG | 61 |
| FASP | 25 |
| Trilinos ML | 23-29 |

# Coupled Poroelasticity on a General Hexahedral Grid

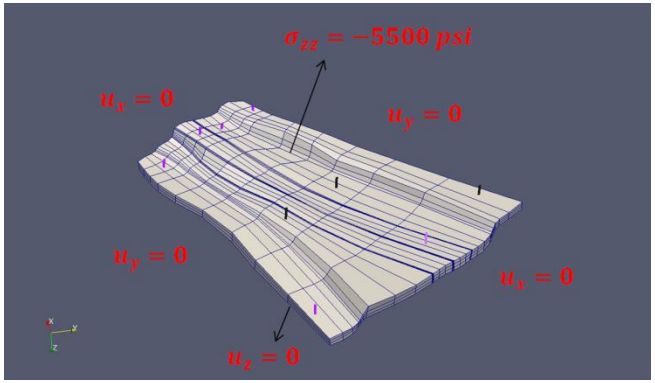
| PARAMETER | QUANTITY | VALUE |
|---|---|---|
| $x$ | x coordinate | $\in$ (-10.4, 8561.6) ft |
| $y$ | y coordinate | $\in$ (68.8, 8822.9) ft |
| $z$ | z coordinate | $\in$ (3796.9, 5436.2) ft |
| $T$ | total simulation time | 7.0 day |
| $\Delta T$ | time step size | 0.1 day |
| $P_0$ | initial pressure | hydrostatic |
| $\rho_0$ | reference fluid density | 56 $lb_m/ft^3$ |
| $\eta$ | fluid viscosity | 1 cp |
| $c_f$ | fluid compressibility | $4.0 \times 10^{-7}$ $psi^{-1}$ |
| $\phi$ | initial porosity | 0.2 |
| $k_{xx}$, $k_{yy}$ | horizontal permeability | $\in$ ($1.0 \times 10^{-15}$, 1592) md |
| $k_{zz}$ | vertical permeability | 0.1 $k_{xx}$ |
| $N_{inj}$ | number of injection wells | 6 |
| $BHP_{inj}$ | bottom hole pressure of injection wells | $\in$ (3300, 4400) psi |
| $N_{prod}$ | number of production wells | 3 |
| $BHP_{prod}$ | bottom hole pressure of production wells | 2000 psi |
| $\sigma_{zz}$ | vertical stress on reservoir top surface | -5500 psi |
| $E$ | Young's modulus | $1.0 \times 10^6$ psi |
| $\nu$ | Poisson's ratio | 0.3 |
| $\rho_s$ | rock density | 165 $lb_m/ft^3$ |



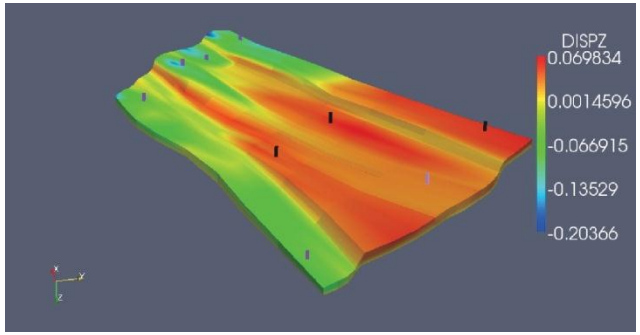

X-permeability Profile




Fluid Pressure at 7.0 Days



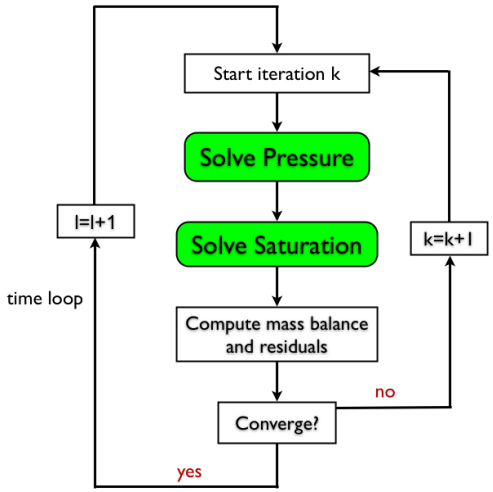

Mechanics Boundary Condition




Vertical Displacement at 7.0 Days

# Modeling Capillarity with the Multipoint Flux Method

- Iterative coupling IMPES Scheme




- Brooks-Corey Capillary Pressure


$$p_c(s_e) = p_d s_e^{-\frac{1}{\lambda}}$$

| Media type | $p_d$ | $\lambda$ |
|---|---|---|
| **type 1** | 135 | 2.49 |
| **type 2** | 37.7 | 3.86 |

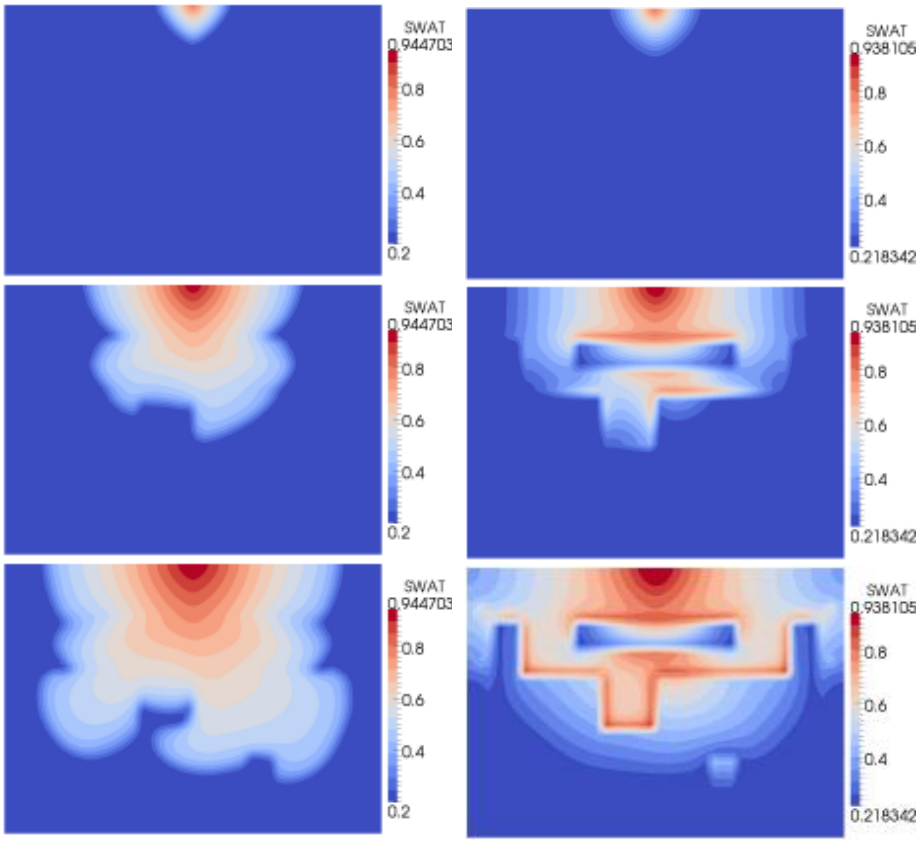
$$s_e = \frac{s - s_{rw}}{1 - s_{rw} - s_{rn}}$$

$k_{rw} = 0.7 s_e^2 \qquad k_{rn} = 0.5(1 - s_e)^2$

$s_{rw} = 0.2 \qquad s_{rn} = 0.05$

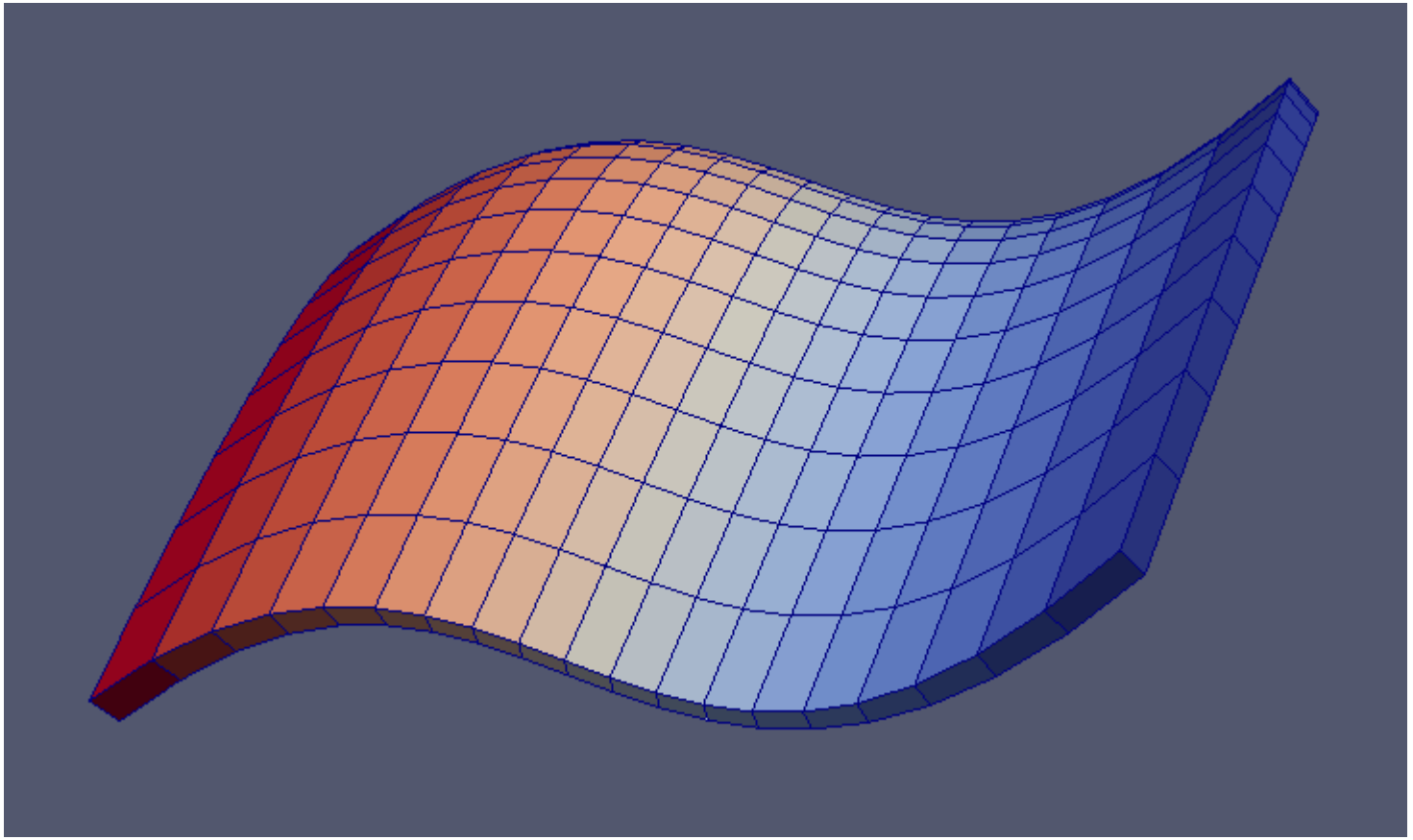
Water Saturation without capillarity

Water Saturation with capillarity



# Using NURBS to represent nonplanar interfaces

Example of using a Non-Uniform Rational B-Splines (NURBS) to represent nonplanar faults and fractures



# References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2. M. F. Wheeler, G. Xue, and I. Yotov, A Family of Multipoint Flux Mixed Finite Element Methods for Elliptic Problems on General Grids, Procedia Computer Science, Vol. 4, 2011, 918-927.
3. M. F. Wheeler, G. Xue, and I. Yotov, Local Velocity Postprocessing for Multipoint Flux Methods on General Hexahedra, ICES REPORT 11-04, The Institute for Computational Engineering and Sciences, The University of Texas at Austin, March 2011, Accepted by International Journal of Numerical Analysis and Modeling.
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